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CALCULATION OF THERMAL STRESSES ARISING IN ELECTRICALLY CONDUCTING MATERIALS WITH THE PASSAGE OF A HIGH-CURRENT PULSE

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A high-current pulse, passing through a rod, whose duration is less than the period of the free vibrations of the rod, sets up thermoelastic vibrations in the rod [1, 2]. A calculation of the stresses arising in a vibrating rod was made in [2], taking account of the temperature dependence of the elastic modulus, under the assumption of a homogeneous distribution of the current.

Neglect of the skin effect in a conductor is not always justified. If the radius of the rod exceeds the thickness of the skin depth, the inhomogeneity of the distribution of the current density must be taken into consideration [3]. An inhomogeneous distribution of the current in a conductor with a high-frequency electrical pulse sets up a temperature gradient and brings about thermal stresses in the material of the conductor [4].

In the present work a calculation is made of the thermal stresses in a sample, subjected to heating by a current pulse, taking account of the skin effect; the statement of the electrodynamic problem coincides with the statement of the problem in [3]. For a description of the process of pulsed heating, the equations of electrodynamics are supplemented by the equations of thermal conductivity and elasticity [5]

$$c_V \frac{\partial T}{\partial t} + \frac{E}{1-2\mu} \frac{\alpha T}{3} \frac{\partial}{\partial t} \frac{\partial u_l}{\partial x_l} = \frac{\partial}{\partial x_l}\left(\varkappa \frac{\partial T}{\partial x_l}\right) + Q(x, t); \tag{1}$$

$$\rho \frac{\partial^2 u_l}{\partial t^2} = \frac{\partial \tau_{lm}}{\partial x_m} + F_l, \quad l, m = 1, 2, 3, \tag{2}$$

where T is the temperature; c_V is the specific heat capacity; E is the Young modulus; μ is the Poisson coefficient; α is the coefficient of volumetric expansion; Q is a function, describing the Joule heating of the sample; τ_{lm} is the tensor of the internal stresses; u_l are the components of the vector of the deformation; ρ is the density; F_l is the component of the volumetric forces acting on the sample.

We consider the quasi-steady-state electrodynamic problem with a constant conductivity σ. The condition of the quasistationary character of the electromagnetic field for conductors with a length of ∼1 m is satisfied for pulses with a duration $\Delta t \geqslant 10^{-6}$ sec. Ohm's law is applied in its simplest form:

$$j_l = \sigma E_l,$$

where j_l is the current density; E_l is the intensity of the electrical field.

In such a statement, the electromagnetic problem is solved separately from the equations of thermal conductivity and elasticity. Under the assumption of the presence of axial symmetry of the problem, the current density has one component differing from zero, $j_z(r, t) = j(r, t)$, for which the equation has the form

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$$\mu_0\sigma \frac{\partial j}{\partial t} = \frac{\partial^2 j}{\partial r^2} + \frac{1}{r}\frac{\partial j}{\partial r}, \quad \mu_0 = 4\pi \cdot 10^{-7}\,\text{H/m}. \tag{3}$$

From the linearity of Eq. (3) there follows the possibility of using the Fourier method for finding the solution. Taking account of the finite nature of the current density at the axis of the conductor, we can obtain an expression for the current density in the form

$$j(r, t) = \frac{1}{2\pi r_0} \int d\omega \, \frac{ikJ_0(ikr)}{J_1(ikr_0)} I(\omega) e^{i\omega t}, \tag{4}$$

where $k^2 = i\mu_0\sigma\omega$; $J_\nu(ikr)$ is a Bessel function of order $\nu(\nu = 0.1)$ of a complex argument; r_0 is the radius of the conductor; $I(\omega)$ is a Fourier expansion of the total current. The total current can have the form of damped vibrations or of an aperiodic signal [3]. Integration of (4) is carried out by the method of residues. For weakly damped vibrations and an aperiodic discharge, (4) coincides with the results obtained in [3]. In the case of a strong skin effect, expression (4) goes over into a well-known exponential distribution of the current density [6].

Article [3] gives a connection between the parameters of the discharge circuit and the parameters of the electrical signal:

$$\beta = \frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}},$$
$$\gamma = \frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}, \tag{5}$$

where R is the active resistance of the circuit; L is the inductance; C is the capacitance. Varying the parameters of the circuit, the parameters of the signal β and γ can be varied. In this case, there is a change in the position of the poles of the function $I(\omega)$ in the complex ω plane. It must be noted that relationships (5) hold for low-frequency pulses (weak skin effect), where the poles of $I(\omega)$ are far from the position for null values of $J_1(ikr_0)$. In the contrary case the connection between the parameters of the signal and the electrical circuit is determined by other relationships. The connection between the parameters of the signal and those of the discharge circuit in the most general case can be obtained from the energy conservation law written for a cylindrical conductor [6].

In the notation $\Omega = ikr_0$, $\Omega_0 = r_0(\mu_0\sigma/\sqrt{LC})^{1/2}$, $\Omega_1 = r_0(\mu_0\sigma R/2L)^{1/2}$, this relationship has the form

$$\Omega^4 - \Omega^3\Omega_1^2 \frac{J_0(\Omega)}{J_1(\Omega)} + \Omega_0^4 = 0. \tag{6}$$

By virtue of the real nature of the expression for the total current, the roots of Eq. (6) have the form of a complex-conjugate pair

$$\Omega_n^{(\pm)} = |\Omega_n| \exp(\pm i\psi_n), \quad n = 0, 1, 2, \ldots.$$

In the case of an aperiodic signal ($\psi_n = \pi/2$) the roots of Eq. (6) satisfy the following relationships:

$\beta_n < \Omega_n < \alpha_n$, where β_n and α_n are determined by the equations $J_0(\beta_n) = 0$, $J_1(\alpha_n) = 0$, $n = 1, 2, 3, \ldots$;

$\Omega_n \to \alpha_n$ with $n \to \infty$. The pulse obtained in a circuit with given parameters represents an infinite series, consisting of an exponent. In the indices of the exponent, the factors with t contain the roots of Eq. (6). For description of the pulse it is sufficient to limit ourselves to the first two terms, since the remaining terms are damped with time far more rapidly than the first two and have only a weak effect on the form of the pulse.

Solution of the problem of the distribution of the current allows of solution of the system of equations of thermal conductivity (1) and elasticity (2).

The equation of the heat balance (1) contains the function $Q = (1/\sigma)j^2(r, t)$, describing the Joule heating of the sample. Durations of the current pulses of 10^{-4}–10^{-6} are considered The phenomenon of thermal conductivity can be neglected, since the characteristic time t_T for this process is great in comparison with the duration of the pulse. For molybdenum samples of cylindrical form with dimensions of $r_0 = 2 \cdot 10^{-3}$ m and length $l = 4 \cdot 10^{-2}$ m, we have $t_T \geqslant 10^{-2}$ sec. The thermoelastic stresses considerably exceed the stresses arising with electrodynamic compression of the sample [1]. The ratio of the thermodynamic stresses τ_0 and the electrodynamic stresses τ_e is on the order of $\tau_e/\tau_0 \leqslant 10^{-4}$.

The duration of the current pulses is more than an order of magnitude greater than the time of the elastic vibrations of the sample. Under these circumstances, the inertial terms make only a small contribution to Eq. (2) and the deformations depend on the time through the temperature. The vector of the deformation u_l in cylindrical geometry, by virtue of the axial symmetry, has two nonzero components u_r and u_z, and there are no torsional deformations, $u = 0$.

Taking account of all the approximations, Eqs. (1), (2) are reduced to the system

$$c_V \frac{\partial T}{\partial t} + \frac{E}{1-2\mu} \frac{\alpha T}{3} \frac{\partial}{\partial t}\left(\frac{\partial u_l}{\partial x_l}\right) = \frac{1}{\sigma} j^2(r, t); \tag{7}$$

$$\frac{\partial^2 u_l}{\partial x_m^2} + \frac{1}{1-2\mu} \frac{\partial^2 u_m}{\partial x_l \partial x_m} - \frac{2(1+\mu)}{1-2\mu} \frac{\alpha}{3} \frac{\partial T}{\partial x_l} = 0 \tag{8}$$

with the following boundary conditions:

1) The absence of heat transfer with the external medium, since the losses of heat by radiation are small in view of the low temperatures, and the evolution of heat with the given durations of the pulses can be neglected;

2) the absence of normal stresses at the lateral surfaces of the rod $\tau_{rr}|_{r = r_0} = 0$;

3) free ends of the rod $\int\limits_0^{r_0} \tau_{zz} r dr = 0$.

The initial conditions are a constant temperature T_0 and the absence of deformation at the moment $t = 0$.

The left-hand part of Eq. (7) contains a term with a vector of the deformation u_l. Since the dependence of the deformation on the temperature is determined by the equilibrium equation (8), the system (7), (8) can be reduced to an equation for the temperature. For the materials under consideration (tantalum, tungsten, molybdenum) the relationship [7] $\alpha E/c_V \sim 1$ is satisfied; therefore, the second term in the right-hand part of Eq. (7) differs from the first term by an amount on the order of (u/r_0). For the case of negligible elastic deformation, we can use the method of successive approximations, expanding the solution in terms of the parameter (u/r_0).

Equations (8) for an axially symmetrical distribution of the temperature are integrated; the components of the vector of the deformation u_r, u_z are expressed in quadratures [7]. The solution of the system (7), (8) and the relationships between the deformations and the stresses [7] make it possible to determine the components of the stress tensor

$$\begin{aligned}
\tau_{rr} &= \frac{E}{1-\mu} \frac{\alpha}{3} \left\{ \frac{1}{r_0^2} \int\limits_0^{r_0} T(r', t) r' dr' - \frac{1}{r^2} \int\limits_0^{r} T(r', t) r' dr' \right\}, \\
\tau_{\varphi\varphi} &= \frac{E}{1-\mu} \frac{\alpha}{3} \left\{ \frac{1}{r_0^2} \int\limits_0^{r_0} T(r', t) r' dr' + \frac{1}{r^2} \int\limits_0^{r} T(r', t) r' dr' - T(r, t) \right\}, \\
\tau_{zz} &= \frac{E}{1-\mu} \frac{\alpha}{3} \left\{ \frac{2}{r_0^2} \int\limits_0^{r_0} T(r', t) r' dr' - T(r, t) \right\}, \\
\tau_{r\varphi} &= \tau_{\varphi z} = \tau_{rz} = 0
\end{aligned} \tag{9}$$

and the distribution of the temperature

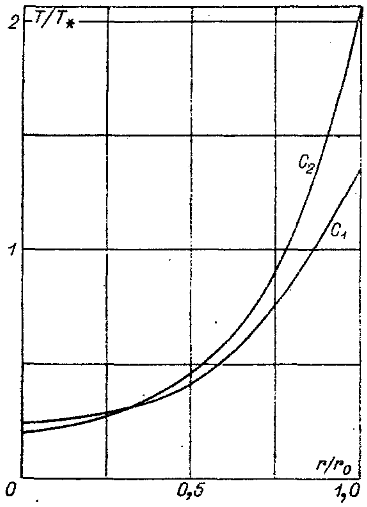

Fig. 1

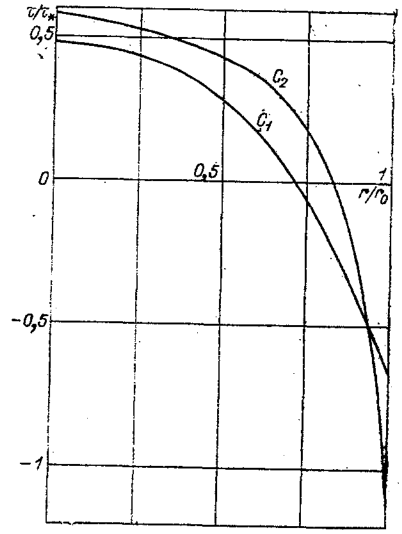

Fig. 2

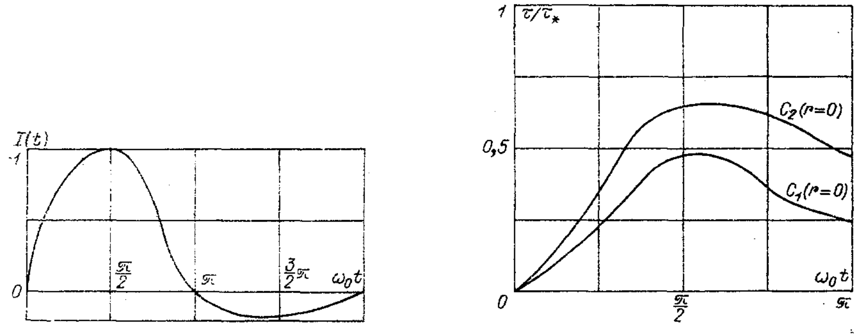

Fig. 3

$$T(r,\ t) = T^{(0)}(r,\ t) + T^{(1)}(r,\ t) + \dots$$

$$T^{(0)}(r,\ t) = \frac{1}{\sigma c_V} \int_0^t j^2(r,\ t')\,dt'; \tag{10}$$

$$T^{(1)}(r,\ t) = \frac{1}{r_0^2} \int_0^t q(r,\ t')\,dt' \int_0^{r_0} r'dr' \int_0^{t'} q(r',\ t'')\,dt'' -$$

$$- \frac{1}{2}\left(\int_0^t q(r,\ t')\,dt'\right)^2 - \frac{1}{r_0^2} \int_0^t q(r,\ t')\,dt' \int_0^{r_0} r'dr' \int_0^t q(r',\ t'')\,dt'', \tag{11}$$

where

$$q(r,\ t) = \frac{1}{\sigma c_V}\, j^2(r,\ t).$$

Using formulas (9)-(11), we can calculate the space and time distribution of the temperature and the associated stressed state of the investigated material.

The distribution of the temperature in the investigated material for the moment of time $\omega_0 t = \pi/2$, where the parameters R, L of the unit are constant while C varies ($C_2 = 0.45C_1$), is shown in Fig. 1. A calculation was made for molybdenum samples $r_0 = 2 \cdot 10^{-3}$ m, $l = 4 \cdot 10^{-2}$ m; the parameters of the unit: $R = 3.4 \cdot 10^{-4}\Omega$; $L = 4.5 \cdot 10^{-7}$H; $C = 3 \cdot 10^{-3}$F; the voltage in the battery of condensers was ~5 kV. The mean temperature is determined according to

$$T_* = \frac{1}{\pi r_0^2}\int_0^{r_0} T(r, t)\, r dr;$$

then $\tau_* = [E/(1-\mu)](\alpha T_*/3)$.

From Fig. 1 it can be seen that with a decrease in the duration of pulse, the temperature gradient rises and along with it there is a rise in the thermal stresses which can lead to fracture (Fig. 2). The thermal stresses attain their maximal value at the end of the firs half of the pulse ($\omega_0 t = \pi/2$), when the total current in the circuit is maximal. With a decrease of the current in the circuit, the temperature at the axis of the rod increases, the temperature gradient decreases, and the internal stresses in the material fall (Fig. 3, I(t) is the total current).

The results of the calculation make it possible 1) to determine the maximal temperature gradient which the material can sustain; 2) in conjunction with experimental data, to obtain the temperature dependence of the heat resistance of the material; 3) to determine the effect of the parameters (R, L, C) of the unit on the character of the temperature distribution in the sample.

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